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Bianchi Type V Magnetized String Dust Bulk Viscous Fluid Cosmological Model with Variable Magnetic Permeability

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Abstract Bianchi Type V magnetized string dust bulk viscous fluid cosmological model with variable magnetic permeability, is investigated. The magnetic field is due to an electric current produced along x-axis. Thus the magnetic fields is in yz-plane and F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic model in terms of cosmic time t, we have also assumed the condition $\zeta\theta =$ constant where ζ the coefficient of bulk viscosity and θ the expansion in the model. The behaviour of the model in presence and absence of magnetic field and bulk viscosity and singularities in the model are also discussed.

Keywords Bianchi V magnetized string dust bulk viscous cosmological model · Variable magnetic permeability

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1 Introduction

Bianchi Type V cosmological models are the natural generalization of FRW (Friedmann-Robertson-Walker) models with negative curvature. These open models are favoured by the available evidences for low density universes (Gott et al. [1]). Bianchi Type V cosmological models have also been studied by number of authors viz. Ftaclas and Cohen [2], Lorentz [3], Roy and Singh [4], Bali and Meena [5], Pradhan and Rai [6], Pradhan et al. [7–9]. The distribution of matter can be satisfactorily described by perfect fluid due to the large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that when neutrino decoupling occurred, the matter behaved like a viscous field in the early stage of the universe. The presence of viscosity in the fluid content has been found to

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explain successfully many physical phenomena in the study of homogeneous cosmological models. The different picture of the universe may appear at the initial state of cosmological evolution due to the dissipative process caused by viscosity as viscosity counteracts the cosmological collapse. Misner [10, 11] has studied the effect of viscosity on the evolution of cosmological models. Several authors viz. Murphy [12], Belinski and Khalatnikov [13], Banerjee et al. [14], Santos et al. [15], Roy and Prakash [16], Padmanabhan and Chitre [17], Johri and Sudarshan [18], Mohanty and Pradhan [19], Beesham [20], Bali et al. [21–23], Pradhan et al. [24] have attempted to find exact solutions of Einstein's field equation by considering viscous effect in isotropic as well as anisotropic models.

Cosmic strings play a significant role in the study of universe. In the early universe (string-dominated era), the strings produce fluctuations in the density of particles. We may speculate that as the strings vanish and particles become important then the fluctuations grow in such a way that finally we shall end up with galaxies. As the strings disappear, the space-time anisotropy introduced by them also disappears. The presence of strings in early universe can be explained using grand unified theories (Kibble [25], Everett [26], Vilenkin [27] and Zel'dovich [28]). These strings have stress energy and they can be classified as geometrical and massive strings. Each massive string is formed by geometric string with particles attached along its extension. Hence the strings that form the cloud, are the generalization of Takabayasi's realistic model of strings [29] that we call p-strings (Letelier [30]). This is the interesting situation wherein we have particles and strings together. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings is due to Letelier [31] who explained that the massive strings are formed by geometric string (Nambu string [32]) with particles attached along its extension. Letelier [33] first used this idea in obtaining some cosmological solutions for massive strings for Bianchi Type I and Kantowski-Sachs space-time. Tikekar et al. [34] have investigated a new class of specific inhomogeneous string cosmological solution for cylindrically symmetric space-times. The present day magnitude of the magnetic energy is very small in comparison with the estimated matter density, it might not have been negligible during early stage of the evolution of the universe. A cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction. Bronnikov et al. [35] have studied the evolution of Bianchi Type I space-time with a global unidirectional electromagnetic field. Melvin [36] in the cosmological solutions for dust and electromagnetic field has argued that for a large part of the history of evolution of the universe, the matter was in a highly ionized state and is smoothly coupled with the field and subsequently forms a neutral matter as a result of universe expansion. Hence the presence of magnetic field in a string dust universe is not unrealistic. The string cosmological models with magnetic field have also been investigated by Banerjee et al. [37], Chakraborty [38], Tikekar and Patel [39, 40], Patel and Maharaj [41], Singh and Singh [42], Singh and Chaubey [43], Wang [44], Bali and Upadhaya [45], Bali and Anjali [46], Bali et al. [47], Bali and Jain [48]. In the above mentioned studies, the magnetic permeability where it is considered, is assumed as constant. Recently Bali [49] has investigated Bianchi Type V magnetized string dust universe with variable magnetic permeability.

In this paper, we have investigated Bianchi Type V magnetized string dust bulk viscous fluid cosmological model with variable magnetic permeability. We assume that current is flowing along x-direction. Thus magnetic field is in yz-plane and F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic model in terms of cosmic time t, we have also assumed the condition $\zeta\theta =$ constant where ζ the coefficient of bulk viscosity and θ the expansion in the model. The physical aspects of the model together with behaviour of the model in presence and absence of magnetic field and bulk viscosity are also discussed.

2 The Metric and Field Equations

We consider Bianchi Type V space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{2x} dz^2 \tag{1}$$

where A, B, C are functions of t-alone.

The energy-momentum tensor (T_i^j) for cloud of string dust is given by Letelier [30] with bulk viscous fluid given by Landau and Lifshitz [50] and Electromagnetic field (E_i^j) given by Lichnerowicz [51] as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \zeta v^\ell_{;\ell}(g_i^j + v_i v^j) + E_i^j \tag{2}$$

where v^i and x^i satisfy the conditions

$$v_i v^i = -x_i x^i = -1, \tag{3}$$

$$v^i x_i = 0 \tag{4}$$

and

$$E_i^j = b\left[|h|^2\left(v_i v^j + \frac{1}{2} g_i^j\right) - h_i h^j\right] \tag{5}$$

with

$$h_i = \frac{\sqrt{-g}}{2b} \in_{ijk\ell} F^{k\ell} v^j \tag{6}$$

where h_i is the magnetic flux vector, b the magnetic permeability, $F^{k\ell}$ the electromagnetic field tensor, $\in_{ijk\ell}$ the Levi-Civita tensor, ζ the coefficient of bulk viscosity, ρ the proper density for a cloud of string with particles attached to them, λ the string tension density, v^i the four velocity of particles, x^i the unit space-like vector representing the direction of string. If the particle density of the configuration is defined by ρ_p then we have

$$\rho = \rho_p + \lambda \tag{7}$$

In a comoving coordinate system, we have

$$v^i = (0, 0, 0, 1), \qquad x^i = \left(\frac{1}{A}, 0, 0, 0\right) \tag{8}$$

We assume that the magnetic field is due to an electric current produced along x-axis. Thus magnetic field is in yz-plane. Therefore F_{23} is the only non-vanishing component of F_{ij} and $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$. We also find that $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conductivity.

Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{9}$$

and

$$F^{ij}_{;j} = 0 \tag{10}$$

are satisfied by

$$F_{23} = H \text{ (constant)} \tag{11}$$

Equation (6) leads to

$$h_1 = \frac{AHe^{-2x}}{bBC} \tag{12}$$

Using (12) in (5), we have

$$E_1^1 = -\frac{H^2 e^{-4x}}{2bB^2C^2} = -E_2^2 = -E_3^3 = E_4^4 \tag{13}$$

Thus Einstein's field equation

$$R_i^j - \frac{1}{2} g_i^j = -8\pi T_i^j \tag{14}$$

for the line-element (1) leads to the following system of equations

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = 8\pi \left(\frac{H^2}{2B^2C^2} + \lambda + \zeta\theta \right), \tag{15}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -8\pi \left(\frac{H^2}{2B^2C^2} - \zeta\theta \right), \tag{16}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -8\pi \left(\frac{H^2}{2B^2C^2} - \zeta\theta \right), \tag{17}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{A^2} = 8\pi \left(\rho + \frac{H^2}{2B^2C^2} \right), \tag{18}$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{19}$$

To get the deterministic model in terms of cosmic time t, we have assumed that magnetic permeability (b) is a variable quantity and assumed as

$$b = e^{-4x} \tag{20}$$

The motivation for assuming magnetic permeability $b = e^{-4x}$ is explained as: The L.H.S. of (15)–(18) are functions of time t-alone while R.H.S. of (15)–(18) are functions of x and t both if (13) is taken into consideration. To avoid such type of discrepancies, the magnetic permeability is assumed as $b = e^{-4x}$ which is physically valid assumption.

$$b \to 0 \quad \text{when } x \to \infty \quad \text{and} \quad b \to 1 \quad \text{when } x \to 0.$$

3 Solution of Field Equations

Zel'dovich [28] in his investigation has explained that $\rho_s/\rho_c \sim 2.5 \times 10^{-3}$ where ρ_s is the mass density and ρ_c the critical density then strings frozen in plasma would change the density like a^{-2} i.e. like t^{-1} in the radiation dominated universe where a is the radius of the universe. In this approximation, strings would soon be dominant and tension along the string (λ) is equal to its energy density (ρ) per unit length and the particle density (ρ_p) of the configuration is zero. Thus for string dust condition, we have

$$\rho = \lambda \tag{21}$$

Equation (19) leads to

$$A^2 = LBC \tag{22}$$

where L is the constant of integration.

To get the deterministic model of the universe, we have assumed two conditions:

(i) string dust condition: $\rho = \lambda$
(ii) $\zeta\theta =$ constant

i.e. the coefficient of bulk viscosity is inversely proportional to the expansion (θ).

Equations (15) and (18) after using string dust condition $\rho = \lambda$, lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{2}{A^2} = \ell \tag{23}$$

where

$$8\pi\zeta\theta = \ell(\text{constant}) \tag{24}$$

and

$$v^{\ell}_{;\ell} = \theta \tag{25}$$

Equations (16) and (17) lead to

$$\frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{2}{A^2} = -\frac{K}{B^2C^2} + 2\ell \tag{26}$$

where

$$K = 8\pi H^2 \tag{27}$$

Now (19) leads to

$$\frac{2A_{44}}{A} = \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{1}{2}\frac{B_4^2}{B^2} - \frac{1}{2}\frac{C_4^2}{C^2} + \frac{B_4C_4}{BC} \tag{28}$$

Using (22) and (28) in (26), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{1}{LBC} = -\frac{K}{2B^2C^2} + \ell \tag{29}$$

Let us assume

$$BC = \mu, \qquad \frac{B}{C} = \nu \tag{30}$$

Using the assumptions given by (30) in (23) and (29), we have

$$\frac{\mu_{44}}{2\mu} - \frac{\mu_4^2}{2\mu^2} + \frac{1}{4}\frac{\nu_4^2}{\nu^2} + \frac{1}{L\mu} = \frac{\ell}{2} \tag{31}$$

and

$$\frac{\mu_{44}}{\mu}-\frac{1}{4}\frac{\mu_4^2}{\mu^2}+\frac{1}{4}\frac{\nu_4^2}{\nu^2}-\frac{1}{L\mu}=-\frac{K}{2\mu^2}+\ell \tag{32}$$

Equations (31) and (32) lead to

$$2\mu_{44}+\frac{1}{\mu}\mu_4^2=\frac{8}{L}-\frac{2K}{\mu}+2\ell\mu \tag{33}$$

Equation (33) leads to

$$f^2=\left(\frac{d\mu}{dt}\right)^2=\frac{2\ell L\mu^3+12\mu^2-6KL\mu+3NL}{3L\mu} \tag{34}$$

where $\mu_4=f(\mu)$, $\mu_{44}=ff'$, $f'=\frac{df}{d\mu}$ and N is the constant of integration.

From (16), (17) and (19), we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=0 \tag{35}$$

Using assumptions (30) in (35), we have

$$\frac{\nu_4}{\nu}=\frac{M}{\mu^{3/2}} \tag{36}$$

M being the constant of integration.

To get the deterministic solution in terms of cosmic time t, we assume $N=0$. Thus (34) leads to

$$\frac{\sqrt{3L}d\mu}{\sqrt{2L\ell\mu^2+12\mu-6KL}}=dt \tag{37}$$

Equation (37) leads to

$$\frac{d\mu}{\sqrt{(\mu+\frac{3}{L\ell})^2-(\frac{3K}{\ell}+\frac{9}{L^2\ell^2})}}=\sqrt{\frac{2\ell}{3}}dt \tag{38}$$

which leads to

$$\mu=\frac{3}{L\ell}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right)-1\right] \tag{39}$$

Using (39) in (36), we have

$$\frac{d\nu}{\nu}=\frac{M(L\ell)^{3/2}dt}{(3)^{3/2}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right)-1\right]^{3/2}} \tag{40}$$

which leads to

$$\nu=\exp\left[\int\frac{M(L\ell)^{3/2}dt}{(3)^{3/2}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right)-1\right]^{3/2}}\right] \tag{41}$$

Hence the metric (1) reduces to the form

$$ds^2 = -dt^2 + \frac{3}{\ell}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right]dx^2$$
$$+ \frac{3}{L\ell}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right]e^{2x}\left[\nu dy^2 + \nu^{-1}dz^2\right] \tag{42}$$

where ν is determined by (41).

In the absence of magnetic field i.e. when $K = 0$ then the metric (42) leads to

$$ds^2 = -dt^2 + \frac{3}{\ell}\left[\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right]dx^2$$
$$+ \frac{3}{L\ell}\left\{\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right\}e^{2x}\{\nu dy^2 + \nu^{-1}dz^2\} \tag{43}$$

where ν is determined by (41) and is given by

$$\log\nu = \frac{ML^{3/2}}{4}\left\{-\left(\frac{2\ell}{6}\right)\frac{\cosh(\frac{\sqrt{\frac{2\ell}{3}}t}{2})}{\sinh^2(\frac{\sqrt{\frac{2\ell}{3}}t}{2})} + \frac{2\ell}{6}\log\coth\left(\frac{\sqrt{\frac{2\ell}{3}}t}{4}\right)\right\} + \log S \tag{44}$$

where S is the constant of integration.

In the absence of bulk viscosity i.e. when $\ell \to 0$ then we have

$$\mu = \frac{3t^2}{\alpha} \tag{45}$$

where $L = \alpha/3$, α being a constant and ν is in absence of bulk viscosity i.e. when $\ell \to 0$ then (44) leads to

$$\nu = Se^{-\frac{ML^{3/2}}{4t}} \tag{46}$$

Hence in the absence of bulk viscosity, the metric (43) reduces to the form

$$ds^2 = -dt^2 + t^2dx^2 + St^2e^{-\frac{ML^{3/2}}{4t}}e^{2x}dy^2 + \frac{t^2}{S}e^{\frac{ML^{3/2}}{4t}}e^{2x}dz^2 \tag{47}$$

4 Some Physical and Geometrical Features

The energy density (ρ), the string tension density (λ), the expansion (θ), the shear (σ), the spatial volume (R^3) and deceleration parameter (q) for the model (42) in the presence of magnetic field and bulk viscosity are given by

$$8\pi\rho = \frac{\ell(KL^2\ell+3)}{6}\frac{\sinh^2(\sqrt{\frac{2\ell}{3}}t)}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^2}$$

$$- \frac{M^2 L^3 \ell^3}{108[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^3} - \frac{L\ell}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]}$$

$$- \frac{KL^2\ell^2}{18[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\sqrt{\frac{2\ell}{3}}t - 1]^2} = 8\pi\lambda, \tag{48}$$

$$\theta = \frac{3}{2}\sqrt{\frac{2\ell}{3}}\frac{\sqrt{\frac{KL^2\ell+3}{3}}\sinh(\sqrt{\frac{2\ell}{3}}t)}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]}, \tag{49}$$

$$\sigma = \frac{1}{2}\frac{M(L\ell)^3}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^{3/2}}, \tag{50}$$

$$\begin{aligned} R^3 &= \sqrt{-g} = \sqrt{L}\mu^{3/2}e^{2x} \\ &= \sqrt{L}\left(\frac{3}{L\ell}\right)^{3/2}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right]^{3/2} e^{2x}, \end{aligned} \tag{51}$$

$$\begin{aligned} q &= -\frac{\ddot{R}/R}{\dot{R}^2/R^2} \\ &= -\frac{2}{\sqrt{\frac{KL^2\ell+3}{3}}}\frac{\cosh(\sqrt{\frac{2\ell}{3}}t)}{\sinh^2(\sqrt{\frac{2\ell}{3}}t)}\left[\sqrt{\frac{KL^2\ell+3}{3}}\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right] \end{aligned} \tag{52}$$

The above mentioned quantities in absence of magnetic field, are given by

$$\begin{aligned} 8\pi\rho &= \frac{\ell}{2}\frac{\sinh^2(\sqrt{\frac{2\ell}{3}}t)}{[\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^2} - \frac{M^2L^3\ell^3}{108[\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^3} \\ &\quad - \frac{L\ell}{[\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]} = 8\pi\lambda, \end{aligned} \tag{53}$$

$$\theta = \frac{(\frac{3}{2})(\sqrt{\frac{2\ell}{3}}\sinh(\sqrt{\frac{2\ell}{3}}t))}{\cosh(\sqrt{\frac{2\ell}{3}}t) - 1}, \tag{54}$$

$$\sigma = \frac{M}{2}\frac{(L\ell)^{3/2}}{[\cosh(\sqrt{\frac{2\ell}{3}}t) - 1]^{3/2}}, \tag{55}$$

$$R^3 = \sqrt{L}\left(\frac{3}{L\ell}\right)^{3/2}\left[\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right]^{3/2} e^{2x}, \tag{56}$$

$$q = -2\frac{\cosh(\sqrt{\frac{2\ell}{3}}t)}{\sinh^2(\sqrt{\frac{2\ell}{3}}t)}\left[\cosh\left(\sqrt{\frac{2\ell}{3}}t\right) - 1\right] \tag{57}$$

In the absence of bulk viscosity i.e. when $\ell \to 0$ then these quantities are given by

$$8\pi\rho = \frac{3-\alpha}{t^2} - \frac{M^2\alpha^3}{108t^6}, \tag{58}$$

$$\theta = \frac{3}{t}, \tag{59}$$

$$\sigma = \frac{M(\alpha)^{3/2}}{2t^{3/2}}, \tag{60}$$

$$R^3 = \left(\frac{3}{\alpha}\right)t^3 e^{2x}, \tag{61}$$

$$q < 0 \quad \text{where } \cosh\left(\frac{2\ell}{3}t\right) > 1. \tag{62}$$

5 Discussion

The equations of state are restricted by the energy conditions given by Hawking and Ellis [52]. The dominant energy condition implies that $\rho \geq 0$ and $\rho^2 \geq \lambda^2$. Thus $\rho \geq 0$ leads to

$$\begin{aligned}
&\left(\frac{KL^2\ell+3}{3}\right)\ell\frac{\sinh^2(\frac{2\ell}{3}t)}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t)-1]^2} \\
&\geq \frac{M^2L^3\ell^2}{108[\sqrt{\frac{KL^2+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t)-1]^3} + \frac{L\ell}{[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\frac{2\ell}{3}t)-1]} \\
&\quad + \frac{KL^2\ell^2}{18[\sqrt{\frac{KL^2\ell+3}{3}}\cosh(\sqrt{\frac{2\ell}{3}}t)-1]}
\end{aligned} \tag{63}$$

The expansion (θ) in the model (42) in presence of magnetic field and bulk viscosity is zero at $t=0$. The expansion in the model increases as time increases. Since shear (σ) is zero for large values of t. Hence the model isotropizes for large values of t. The spatial volume increases as time increases. The deceleration parameter $q<0$ where $\cosh(\sqrt{\frac{2\ell}{3}}t) > 1$. Hence the model (42) represents an accelerating universe in presence of magnetic field and bulk viscosity.

The energy condition $\rho \geq 0$ in absence of magnetic field, leads to

$$\frac{\ell\sinh^2(\sqrt{\frac{2\ell}{3}}t)}{2[\cosh^2(\sqrt{\frac{2\ell}{3}}t)-1]^2} \geq \frac{M^2L^3\ell^3}{108[\cosh(\sqrt{\frac{2\ell}{3}}t)-1]^3} + \frac{L\ell}{[\cosh(\sqrt{\frac{2\ell}{3}}t)-1]} \tag{64}$$

The model (43) in absence of magnetic field starts with a big-bang at $t=0$ and the expansion in the model decreases as time increases. Since $\sigma = 0$ for large values of t. Hence the model (43) isotropizes for large values of t. The spatial volume (R^3) increases as time increases. Since $q<0$ where $\cosh(\sqrt{\frac{2\ell}{3}}t) > 1$. Hence the model (43) represents an accelerating universe. The model (43) has Point Type singularity at $t=0$ (MacCallum [53]).

The energy condition $\rho \geq 0$ in absence of magnetic field and bulk viscosity for the model (47) leads to

$$\left[\frac{M\alpha^{3/2}}{\sqrt{108(3-\alpha)}}\right]^{1/2} \leq t \tag{65}$$

Thus the model (47) exists during the span of time given by (65). The model (47) starts with a big-bang at $t = 0$ and the expansion in the model decreases as time increases. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$, hence the model (47) isotropizes for large values of t. The spatial volume (R^3) increases as time increases. Since the deceleration parameter $(q) < 0$. Hence the model (47) represents an accelerating universe where $\cosh(\sqrt{\frac{2\ell}{3}}t) > 1$. The model (47) has Point Type singularity at $t = 0$ (MacCallum [53]).

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